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/*
 * tersest_triangulation.h
 *
 * This data structure -- which is really just a formatting convention for
 * an array of chars -- compresses the information in a TerseTriangulation
 * to the maximum extent possible. It works only for Triangulations of 7
 * or fewer Tetrahedra. It is intended for use in storing libraries of
 * manifolds, such as the 5-, 6- and 7-tetrahedron censuses.
 *
 * Here's how the information from the TerseTriangulation data structure
 * is stored. The description is for a 7-tetrahedron Triangulation.
 * With fewer Tetrahedra some bits will be unused, but we always use
 * the same number of bytes. (The reason we always use the same number
 * of bytes is so that we can retrieve a manifold from the middle of a long
 * file without reading the whole file.) Recall that a TerseTriangulation
 * for n Tetrahedra has a glues_to_old_tet[] array of length 2n, and
 * which_old_tet[] and which_gluings[] arrays of length n+1. So for
 * 7 Tetrahedra, glues_to_old_tet[] has length 14, while which_old_tet[]
 * and which_gluings[] each have length 8.
 *
 * num_tetrahedra
 *
 * is redundant and is not stored at all. The number of Tetrahedra can
 * be deduced from the glues_to_old_tet array by keeping track of how
 * many free faces are available at each stage.
 *
 * glues_to_old_tet[]
 *
 * is stored in the first two bytes of the TersestTriangulation string.
 *
 * glues_to_old_tet[0] is stored in the lowest-order bit of byte 0,
 * . . . and so on until . . .
 * glues_to_old_tet[7] is stored in the highest-order bit of byte 0.
 *
 * glues_to_old_tet[8] is stored in the lowest-order bit of byte 1,
 * . . . and so on until . . .
 * glues_to_old_tet[13] is stored in the sixth bit of byte 1.
 *
 * Note that the two highest-order bits in byte 1 remain available for
 * other purposes. The highest-order bit tells whether a Chern-Simons
 * invariant is present (cf. below).
 *
 * which_old_tet[]
 * which_gluings[]
 *
 * which_old_tet[] and which_gluings[] are stored in tandem.
 *
 * which_old_tet[i] is stored in the three highest-order bits
 * of byte 2+i. which_old_tet[i] will always be an integer
 * in the range [0, 6], so it fits comfortably in three bits.
 *
 * which_gluings[i] is stored in the five lowest-order bits
 * of byte 2+i. Each Permutation is converted to an integer
 * in the range [0, 23], which fits comfortably in five bits.
 * The array index_by_permutation[] in tables.c does the conversion.
 *
 * CS_is_present
 * CS_value
 *
 * CS_is_present is stored in the highest-order bit of byte 1.
 *
 * if (CS_is_present == TRUE)
 *     bytes 10 through 17 hold the CS_value, encoded as follows.
 *     First add a multiple of 1/2, if necessary, so that the CS_value
 *     lies in the range [-1/4, 1/4). Then multiply by 2 and add 1/2
 *     so that it lies in the range [0, 1). The CS_value is a
 *     double which on some machines (e.g. the Mac) has an 8-byte
 *     mantissa. In binary, this is a number like 0.100101000..., with
 *     64 meaningful binary digits after the decimal point (the "binary
 *     point" ?). The first 8 digits are stored in byte 10 of the
 *     TersestTriangulation string, the next 8 digits are stored in
 *     byte 11, and so on until the final 8 meaningful digits are stored
 *     in byte 17. On some machines (e.g. on most UNIX workstations)
 *     doubles have only 6-byte mantissas, in which case we just

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*      zero-fill bytes 16 and 17 of the TersestTriangulation string
*      (in practice this shouldn't occur, because the libraries of
*      manifolds will be created on the Mac at full accuracy, and then
*      only read on less accurate platforms).
*
*      if (CS_is_present == FALSE)
*          bytes 10 through 17 are unused.
*/

#ifndef _tersest_triangulation_
#define _tersest_triangulation_

typedef unsigned char TersestTriangulation[18];

#endif
```